A Novel Approach to Perform Rank-one Updates for Machine Learning

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Introduction

- Computational errors encountered in Machine Learning (ML) can lead these algorithms astray.
- Roundoff errors due to fixed-precision computations have been given little attention.
- For the most part, these errors are associated with solving System of Linear Equations (SLE).
- Transforming ML algorithms into unlimited precision is computationally expensive.
- Rank-one updates provide computational savings by taking advantage that the SLEs solved are close to each other.
- There is a need to perform these updates without any errors.

Objective

The goal is to develop an algorithm which performs Rank-one updates which is not only computationally fast but is also free from rounding errors.

Idea

- The Round-off Error Free (REF) framework is founded on the Integer Preserving Gaussian Elimination algorithm.
- This framework is used to solve rational SLEs using integer arithmetic without any rounding errors.
- In this framework, there is a REF LU factorization that can be efficiently computed to solve SLEs.
- Our main idea is to update the factorization coefficients for the types of changes that are encountered in ML algorithms.

Methodology and Results

- Building the REF-LU factorization from scratch takes O(n^3) operations; hence, we focus on developing an update algorithm that takes only O(n^2) operations without any rounding errors.
- Let A be a $(n \times n)$nonsingular matrix whose factorization is known, the Rank-one update of A is defined as,

Rank-one Update

REF-LU(A) → **REF-LU(B)**

$$B = A + \sigma\boldsymbol{\gamma}\boldsymbol{\gamma}^T$$

σ is an integer scalar

$\boldsymbol{\gamma}$ is a column vector with n entries

Example

$$\underset{A}{\begin{bmatrix} 2 & 7 & 3 \\ 5 & 4 & 9 \\ 1 & 6 & 8 \end{bmatrix}} + \underset{\sigma}{1} * \underset{\boldsymbol{\gamma}}{\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}} * \underset{\boldsymbol{\gamma}^T}{\begin{bmatrix} 4 & 1 & 3 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 18 & 11 & 15 \\ 9 & 5 & 12 \\ 13 & 9 & 17 \end{bmatrix}}$$

Computational Results

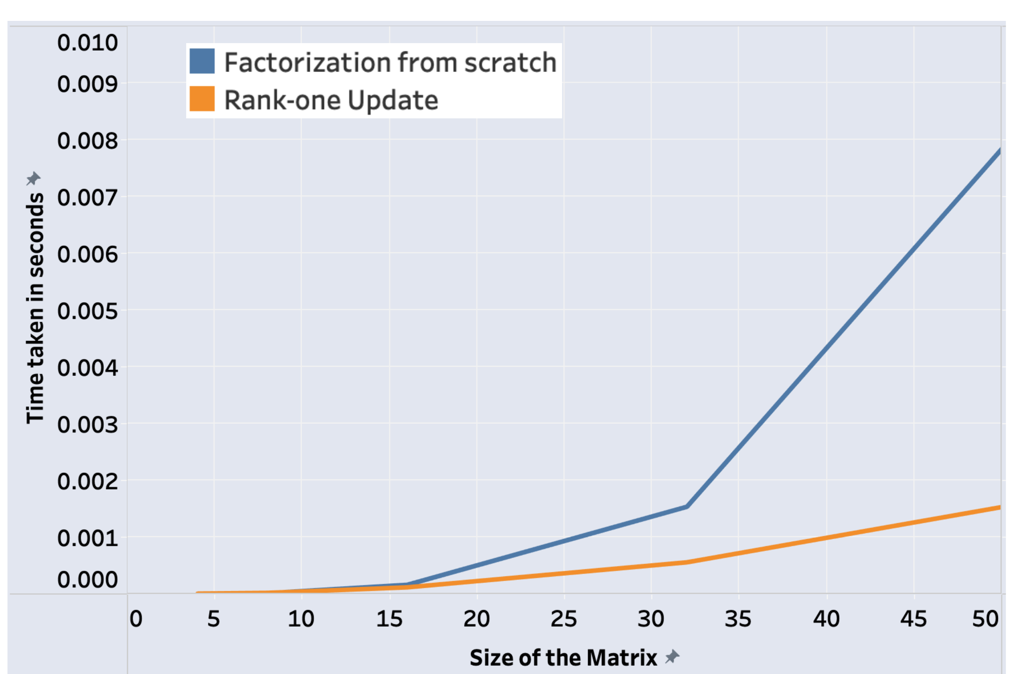
Size of the matrix	Time taken in seconds	
	From scratch	Rank-one update
4	2.06E-6	6.41E-06
8	1.33E-05	1.96E-05
16	1.61E-04	1.17E-04
32	1.54E-03	5.58E-04
64	0.0127	0.002
128	0.15	0.014
256	2.54	0.128
512	58.50	1.43
1024	1463	17.6

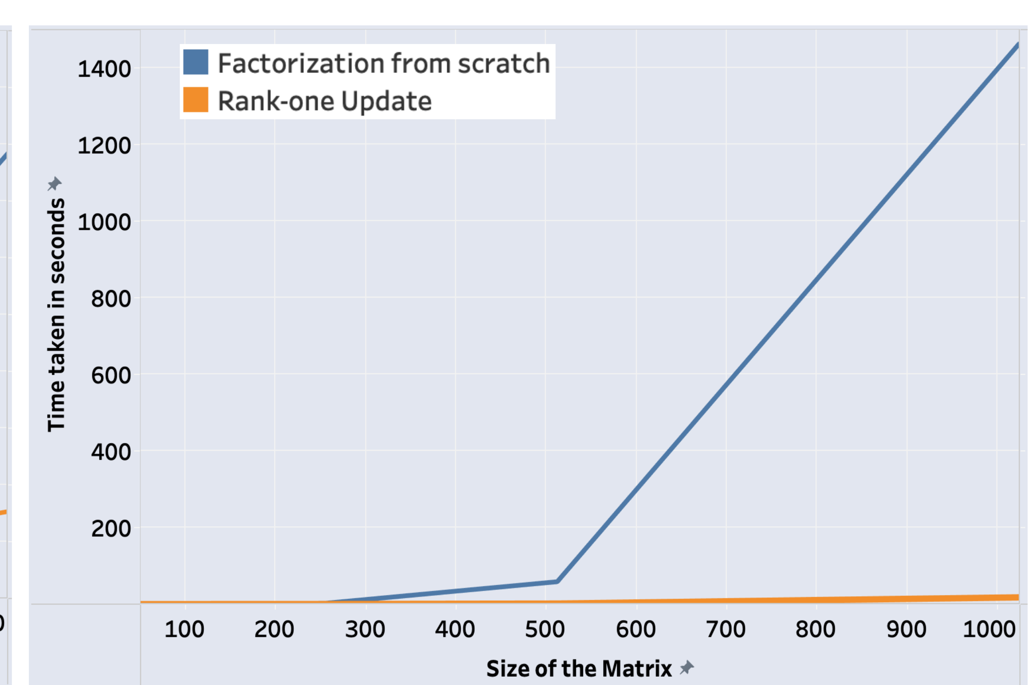
We have devised an update algorithm that has been tested numerically.

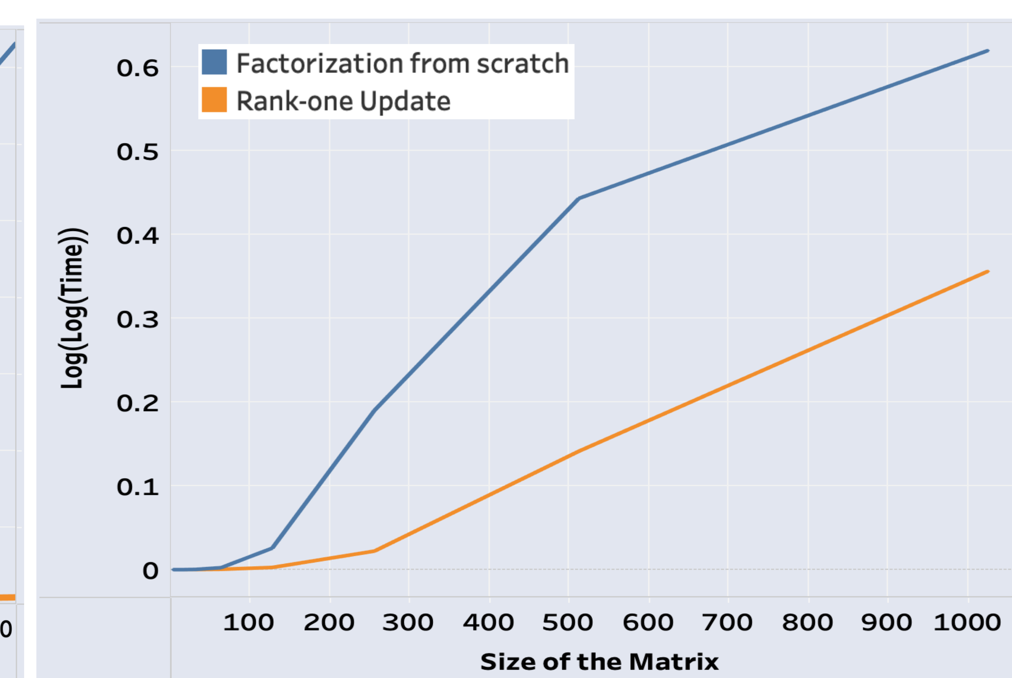
Highlight #1 Initially when the size of the matrix (N) is small (less than 15), there is not much computational difference between our method and factorizing B from scratch.

Highlight #2 When size increases, the time taken to perform the factorization from scratch increases very fast.

Highlight #3 The third graph represents a log log scale of the time taken and from this it is evident that performing a Rank-one update is faster than performing factorization from scratch.







Implementation Details

- Nine different matrices of size ranging from 2^2 to 2^{10} were used to perform computations.
- The elements of the matrix were selected randomly from the numbers -10 to +10.
- Each size was tested with 32 different seeds.
- The time statistics for each size of the matrix was calculated by averaging the 32 different seeds
- To perform high precision operations, the algorithm was programmed in C++ using the GNU GMP library .
- The computations were performed via the ASU Research Computing Cluster

Future Work

- Code other algorithms and compare the results.
- Generalize this approach to perform other low rank updates.

References

- Escobedo, A. R., Moreno-Centeno, E., & Lourenco, C. (2018). Solution of dense linear systems via roundoff-error-free factorization algorithms: Theoretical connections and computational comparisons. *ACM Transactions on Mathematical Software (TOMS)*, *44*(4), 1-24.
- Escobedo, A. R., & Moreno-Centeno, E. (2017). Roundoff-error-free basis updates of LU factorizations for the efficient validation of optimality certificates. *SIAM Journal on Matrix Analysis and Applications*, *38*(3), 829-853.
- Escobedo, A. R., & Moreno-Centeno, E. (2015). Roundoff-error-free algorithms for solving linear systems via Cholesky and LU factorizations. *INFORMS Journal on Computing*, *27*(4), 677-689.

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